Topics in Philosophy of Mathematical Practice 2020
`https://kurser.ku.dk/course/nndk19000u/2019-2020`

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Contents

1 Introduction

The summer course *Topics in Philosophy of Mathematical Practice* offers an introduction to the establishing field of practice-oriented, empirically informed philosophy of mathematics taught by expert researches in the field.

Students are expected to have *either* a background in mathematics (equivalent to a BSc) and an interest in philosophy (of mathematics) *or* a background in philosophy (equivalent of a BA) and an interest in mathematics and philosophy of science. Specifically, basic knowledge at the level of the textbook *Invitation til matematikkens videnskabsteori* (Johansen and Sørensen, 2014) will be expected about the main foundational positions in mathematics (especially logicism and formalism), heuristics of problem choice and problem solving, Lakatos' *Proofs and Refutations*, and the social organization of science including peer review.

The course consists of an online (asynchronous) part and a synchronous part. These will focus on four core topics of PMP:

1. Proofs as vehicles of communication in mathematical practice

2. The role of diagrams in mathematical practice

3. Experimental (non-deductive) aspects of mathematical practice

4. Formal aspects of mathematical practice

We will use and discuss qualitative methods (interviews, observations, case studies) and quantitative methods (corpus analysis, questionaires).

During the synchronous part, students will begin work on their individual projects. They will develop their ideas based on supervision and peer-feedback. Two sessions are devoted to progress-presentations and discussion of projects (see below).

Reading assignments will (mostly) include DOIs which should enable direct download from the UCPH network or through the Royal Library (`https://soeg.kb.dk`). A compendium of the links and remaining texts is posted on `Absalon`.

The synchronous part will be taught using Zoom.

Using `slack`

The entire course, but especially the asynchronous part, will be highly student-driven. *The course will only be as good as the students make it.* To facilitate lively discussion, we have created a forum on the platform `slack` (`https://tpmp2020.slack.com/`). You can access `slack` either in a browser or in apps for iPhone and Android (see `https://slack.com/intl/en-dk/downloads/`).

You will soon receive an invitation to the `slack` workspace through your official KU mail. We know that many students have problems accessing their KU mail at the moment, so if you wish to be invited through another mail address, please contact Mikkel (mwj@ind.ku.dk) as soon as possible.

The `slack` forum contains a general channel as well as channels dedicated to discussing the readings and assignments of the online weeks. Students are expected to read and discuss the materials posed and engage with each other in a constructive development of ideas (in channels such as `#week1-material`). Students will post their (preliminary) assignments by the end of the week (in channels such as `#week1-assignment`) and use time the following week to discuss each other's assignments in those channels, helping to improve ideas, content, and presentation. The final version of the assignment is handed it at the very end of the course as part of the deliverables together with the indvidual report (see below). The teachers will follow the discussions on `slack` with great interest but only moderate them to point out grave mistakes.

2   Online (asynchronous) part

For the online part of the course, we have assigned texts for reading and dicussion as well as a small, mandatory weekly exercise which will help you prepare for the synchronous part.

2.1   Online week 1 (July 13 to July 17): Observation studies

Reading

For the first online week, we will be reading Hamami and Morris (2020) as a general introduction to the field of Philosophy of Mathematical Practice (PMP). We have provided a vodcast which can serve as an introduction (*Welcome to TPMP: Introduction to the Philosophy of Mathematical Practice* 2019). In addition to the philosophical themes introduced therein, you may also consider why PMP would be relevant for practicing mathematicians or to mathematics educators.
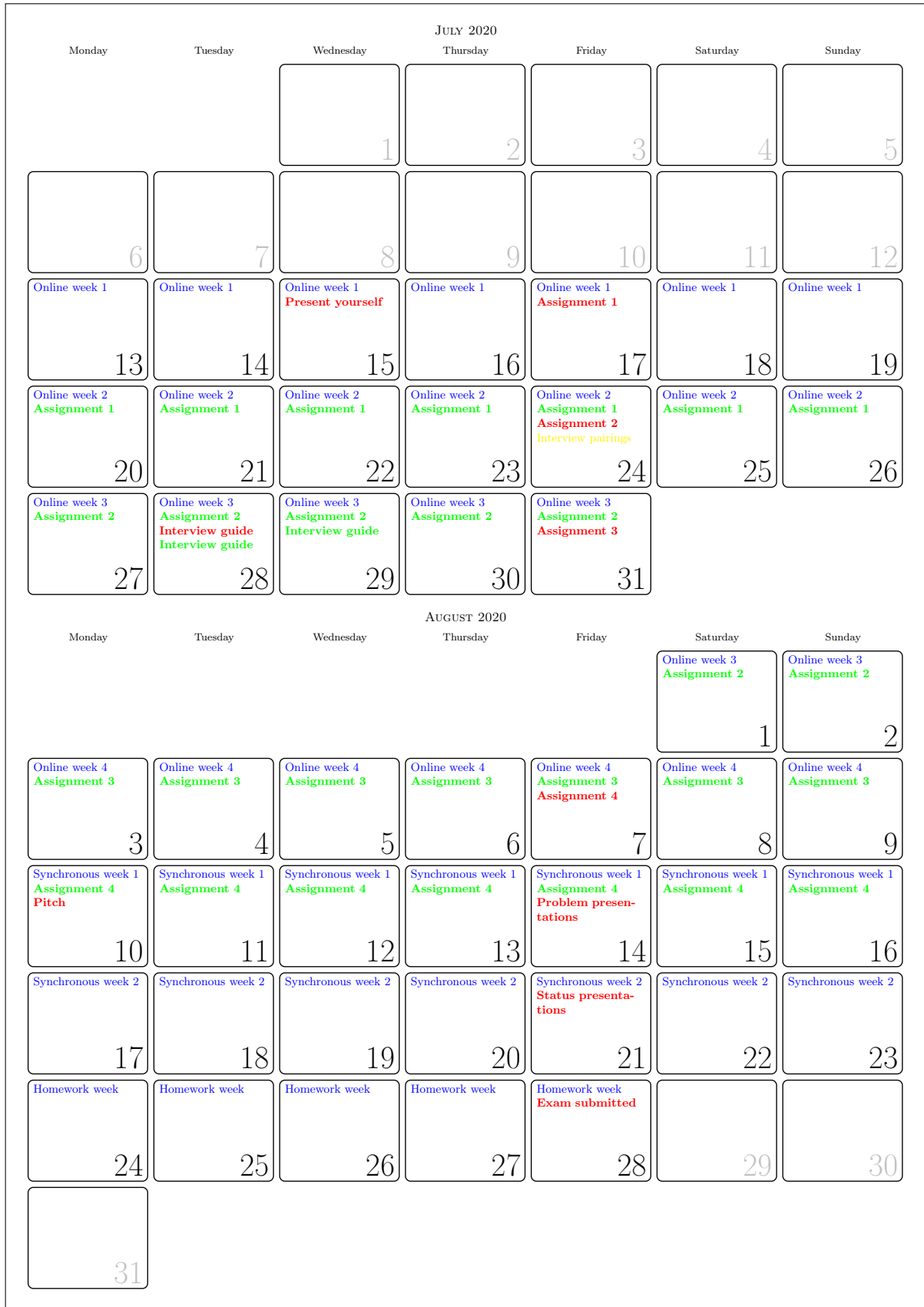
July 2020

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
		1	2	3	4	5
6	7	8	9	10	11	12
Online week 1 — 13	Online week 1 — 14	Online week 1 / Present yourself — 15	Online week 1 — 16	Online week 1 / Assignment 1 — 17	Online week 1 — 18	Online week 1 — 19
Online week 2 / Assignment 1 — 20	Online week 2 / Assignment 1 — 21	Online week 2 / Assignment 1 — 22	Online week 2 / Assignment 1 — 23	Online week 2 / Assignment 1 / Assignment 2 / Interview pairings — 24	Online week 2 / Assignment 1 — 25	Online week 2 / Assignment 1 — 26
Online week 3 / Assignment 2 — 27	Online week 3 / Assignment 2 / Interview guide / Interview guide — 28	Online week 3 / Assignment 2 / Interview guide — 29	Online week 3 / Assignment 2 — 30	Online week 3 / Assignment 2 / Assignment 3 — 31		

August 2020

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
					Online week 3 / Assignment 2 — 1	Online week 3 / Assignment 2 — 2
Online week 4 / Assignment 3 — 3	Online week 4 / Assignment 3 — 4	Online week 4 / Assignment 3 — 5	Online week 4 / Assignment 3 — 6	Online week 4 / Assignment 3 / Assignment 4 — 7	Online week 4 / Assignment 3 — 8	Online week 4 / Assignment 3 — 9
Synchronous week 1 / Assignment 4 / Pitch — 10	Synchronous week 1 / Assignment 4 — 11	Synchronous week 1 / Assignment 4 — 12	Synchronous week 1 / Assignment 4 — 13	Synchronous week 1 / Assignment 4 / Problem presentations — 14	Synchronous week 1 / Assignment 4 — 15	Synchronous week 1 / Assignment 4 — 16
Synchronous week 2 — 17	Synchronous week 2 — 18	Synchronous week 2 — 19	Synchronous week 2 — 20	Synchronous week 2 / Status presentations — 21	Synchronous week 2 — 22	Synchronous week 2 — 23
Homework week — 24	Homework week — 25	Homework week — 26	Homework week — 27	Homework week / Exam submitted — 28	29	30
31						

Figure 1: Course calendar with main tasks (blue: week; yellow: information on `slack`; red: deadline for upload to `slack`; green: feedback task).

We will then also be reading Rav (1999) which is a semi-classic text setting out many of the problems which the PMP-approach to studying proofs and proving have later picked up on. In continuation of this more traditional approach, we will read Barany and MacKenzie (2014) as an example of the kinds of insights which observation studies of mathematical practice can yield.

The final text for this first week is a methodological introduction to observation studies (Atkinson and Hammersley, 2007). This text is largely a 'how-to' guide on designing an observational study. It will be of very direct relevance to those of you who contemplate doing such an observational study. The rest of you can read the text more casually. You can see the text's advice on method and choice as a source to the epistemic status of observational studies. It is crucial to have this basic understanding of the level of contingency and choice involved in ethnographic work before you read and assess such work. As you might remember from your philosophy of science course, all empirical sciences have similar limitations (to a greater or lesser degree).

Vodcast

Watch the vodcast with Mikkel and Henrik about the field of PMP (*Welcome to TPMP: Introduction to the Philosophy of Mathematical Practice* 2019).

Preparation

1. Make a short video presenation of yourself. Make sure to include information about where and what you study. If relevant, also include something about your interests in and prior experiences related to philosophy of mathematics. Post it on `slack` in the channel `#general` as your first post — and no later than Friday July 17.

2. When reading the texts, if you run into questions or have difficulties understanding, post your questions on `slack` in the channel `#week1-material` — and help each other out by engaging constructively (see also our `slack`-policy, above).

Assignment

3. What would be interesting questions to answer by observing mathematical practice? Formulate one or two suggestions, think about how one could possibly go about answering them.

4. And think about any ethical issues that might arise (see `https://tinyurl.com/y2wfrl7h`).

5. Your suggestions must be posted on `slack` (in the channel `#week1-assignment`) by Friday July 19. Please use the forum to engage with suggestions from others and develop ideas (both pre- and post-submit).

References for online week 1

Atkinson, Paul and Martyn Hammersley (2007). "Research design. Problems, cases and samples". In: *Ethnography. Principles in practice*. 3rd ed. London and New York: Routledge, pp. 20–40.

Barany, Michael J. and Donald MacKenzie (2014). "Chalk. Materials and Concepts in Mathematics Research". In: *Representation in Scientific Practice Revisited*. Ed. by Catelijne Coopmans, Janet Vertesi, Michael Lynch, and Steve Woolgar. Cambridge and London: MIT Press. Chap. 6, pp. 107–129.

Hamami, Yacin and Rebecca Lea Morris (May 2020). "Philosophy of mathematical practice. A primer for mathematics educators". *ZDM Mathematics Education*. DOI: `10.1007/s11858-020-01159-5`.

Rav, Yehuda (Feb. 1999). "Why do we prove theorems?" *Philosophia Mathematica (3)*, vol. 7, no. 1, pp. 5–41. DOI: `10.1093/philmat/7.1.5`.

Welcome to TPMP: Introduction to the Philosophy of Mathematical Practice (2019). URL: `https://youtu.be/r3KyJA-TIRc`.

2.2 Online week 2 (July 20 to July 24): Case studies

Reading

For the second online week, we will focus on the role of representations in mathematical practice and read an interesting case study. The background is set in the semi-classic paper Giaquinto (2011) and in the podcast. We will then read the case study by De Toffoli and Giardino (2014). As a methodological discussion of case studies, we will read Flyvberg (2006), but you could also look at Chang (2012) if you are interested in the role of case studies in (history and) philosophy of science and mathematics.

Podcast

Listen to the podcast by Mikkel on diagrams in mathematical practice and the podcast by Henrik on using case studies in history and philosophy of mathematics. Also watch the vodcast interview with professor Silvia De Toffoli from the *16th International Congress on Logic, Methodology and Philosophy of Science and Technology*, Prague, August 5–10, 2019.

Preparation

1. When reading the texts, if you run into questions or have difficulties understanding, post your questions on `slack`— and help each other out by engaging constructively (see also our `slack`-policy, above).

2. Discuss — with each other on `slack`— what kinds of roles diagrams can possibly play in mathematical proofs. Make suggestions, give examples, and (perhaps) work towards classifications.

Assignment

You can find examples of diagrams either in the historical overview by Milnor (2015) or in some of the mathematical research articles Pragacz and Ratajski (1996), Francesco (2000), Vogtmann (2014), Buch (2015), and Drasin and Pankka (2015), which are particularly rich in non-linear elements such as diagrams. Since these are front-end research papers, you will probably not be able to follow the mathematics.

3. Choose some examples of diagrams in these papers and try to describe them and their use as far as you understand or believe you understand them. Feel free to coordinate dividing the examples between you, if you like.

4. Think of questions such as: What role(s) do these diagrams play? How are they introduced? What do they consist of? How do they relate to the rest of the argument?

5. Write up a short (approximately 2 pages) report on your findings and post it on `slack` by Friday. Please use the forum to engage with suggestions from others and develop ideas (both pre- and post-submit).

References for online week 2

Buch, Anders (July 2015). "Mutations of puzzles and equivariant cohomology of two-step flag varieties". *Annals of Mathematics*, pp. 173–220. DOI: `10.4007/annals.2015.182.1.4`.

Chang, Hasok (2012). "Beyond Case-Studies: History as Philosophy". In: *Integrating History and Philosophy of Science*. Ed. by S. Mauskopf and T. Schmaltz. Boston Studies in the Philosophy of Science 263. Springer Science. Chap. 8, pp. 109–124. DOI: `10.1007/978-94-007-1745-9_8`.

De Toffoli, Silvia and Valeria Giardino (Aug. 2014). "Forms and Roles of Diagrams in Knot Theory". *Erkenntnis*, vol. 79, no. 4, pp. 829–842. DOI: `10.1007/s10670-013-9568-7`.

Drasin, David and Pekka Pankka (2015). "Sharpness of Rickman's Picard theorem in all dimensions". *Acta Mathematica*, vol. 214, no. 2, pp. 209–306. DOI: `10.1007/s11511-015-0125-x`.

Flyvberg, Bent (2006). "Five Misunderstandings About Case-Study Research". *Qualitative Inquiry*, vol. 12, no. 2, pp. 219–245. DOI: `10.1177/1077800405284363`.

Francesco, P. Di (Apr. 2000). "Folding and coloring problems in mathematics and physics". *Bulletin of the American Mathematical Society*, vol. 37, no. 03, pp. 251–308. DOI: `10.1090/s0273-0979-00-00870-3`.

Giaquinto, Marcus (2011). "Crossing curves: A limit to the use of diagrams in proofs". *Philosophia Mathematica*, vol. 19, no. 3, pp. 281–307. DOI: `10.1093/philmat/nkr023`.

Milnor, John (July 2015). "Topology through the centuries. Low dimensional manifolds". *Bulletin of the American Mathematical Society*, vol. 52, no. 4, pp. 545–584. DOI: `10.1090/bull/1507`.

Pragacz, Piotr and Jan Ratajski (1996). "A Pieri-type theorem for Lagrangian and Orthogonal Grassmannians". *Journal für die reine und angewandte Mathematik*, vol. 476, pp. 143–189. URL: `https://gdz.sub.uni-goettingen.de/id/PPN243919689_0476`.

Vogtmann, Karen (Aug. 2014). "On the geometry of Outer space". *Bulletin of the American Mathematical Society*, vol. 52, no. 1, pp. 27–46. DOI: `10.1090/s0273-0979-2014-01466-1`.

2.3 Online week 3 (July 27 to July 31): Interview studies

Reading

For the third online week of the course, we will focus on interview studies and read an innovative combination of observations and interviews.

For a methodological background to this week and the next, you will read Löwe and Kerkhove (2019).

As an introduction to interview studies, read Brinkmann and Kvale (2015, chap. 7); if you are going to rely on interviews for your own project, please also read Brinkmann and Kvale (2015, chap. 6).

The case study is the paper Kaufman (2016) which relies on longitudinal observations and interviews of a mathematical collaboration.

Podcast and vodcast

Listen to the podcast with Henrik on mathematical objects and concepts from a Lakatosian perspective and the vodcast where Mikkel interviews Stine Adrian about designing and conducting interviews (*Dr. Adrian on qualitative interviews* 2019).

Preparation

1. When reading the methodological texts (Löwe and Kerkhove, 2019; Brinkmann and Kvale, 2015), if you run into questions or have difficulties understanding, post your questions on `slack`— and help each other out by engaging constructively (see also our `slack`-policy, above).

2. Discuss — with each other on `slack`— how the mathematical objects are shaped during the collaboration analysed in Kaufman (2016). How would you characterize the phases in the development of mathematical objects.

Assignment

For this week's assignment, you are all going to conduct short interviews with each other about *your* mathematical practice based on an interview guide which you develop. We will provide the pairings before Friday July 24. Your interviewee's exposure to and experience with mathematics might be quite different from your own, so consider that when planning your interview.

3. Prepare an interview guide. Think about what you want to research; you are quite free to choose. The interview guide must include: a) A question about actual practice ("how do you go about doing X?"), b) A question about the interviewee's analysis ("why do you do Y?"), and c) A question about the interviewee's interpretation ("what do you think about doing Z?"). Upload the interview guide to `slack` (in the channel `#week3-assignment`) no later than Tuesday morning.

4. Arrange for the interview to take place. Think about how you communicate (will you meet in person or use your preferred online platform or telephone?).

5. Comment on each other's interview guides on `slack` no later than Wednesday.

6. Conduct and record an interview of 10–15 minutes. Mind the time! It flies when you talk, but it is hard work transcribing an interview.

7. Transcribe the interview.

8. Write up a short reflection (1–2 pages) and post it on `slack` *together with the final interview guide and the transcription* by Friday. Please use the forum to engage with suggestions from others and develop ideas (both pre- and post-submit).

References for online week 3

Brinkmann, Svend and Steinar Kvale (2015). *InterViews. Learning the Craft of Qualitative Research Interviewing*. 3rd ed. Los Angeles et al.: SAGE.
Dr. Adrian on qualitative interviews (2019). URL: `https://youtu.be/hBQ5YSWI2CE`.
Kaufman, Stav (2016). "On the Emergence of a New Mathematical Object". In: *Mathematical Cultures. The London Meetings 2012–2014*. Ed. by Brendan Larvor. Trends in the history of science. Birkhäuser, pp. 91–110. DOI: `10.1007/978-3-319-28582-5_6`.
Löwe, Benedikt and Bart Van Kerkhove (2019). "Methodological Triangulation in Empirical Philosophy (of Mathematics)". In: *Advances in Experimental Philosophy of Logic and Mathematics*. Ed. by Andrew Aberdein and Matthew Inglis. Advances in Experimental Philosophy. Bloomsbury. Chap. 2, pp. 15–37.

2.4 Online week 4 (August 3 to August 7): Quantitative studies and corpus analysis

Reading

For the fourth and final online week of the course, we will focus on quantitative studies of mathematicians' practices and views about mathematics.

The study Inglis and Aberdein (2014) reports on an empirical investigation into aesthetic views of practicing mathematicians. The study thus adds an empirical perspective on an established discussion in the philosophy of mathematics (and mathematical practice) with key contributions by Rota, Kitcher, Montano and others.

The paper Pease, Aberdein, and Martin (2019) reports on a corpus analysis of an online collaboration, and they focus on types and roles of explanations in mathematics. Thus, this paper is similarly taking up an interesting discussion in both philosophy and didactics of mathematics.

For a methodological introduction to designing questionaire studies, read Saris and Gallhofer (2014, 4–12 and chap. 3); if you plan to use questionaires in your own project, please also read Saris and Gallhofer (2014, chap. 4).

Vodcast

Watch the vodcast interviews with professors Matthew Inglis and Brendan Larvor from the *16th International Congress on Logic, Methodology and Philosophy of Science and Technology*, Prague, August 5–10, 2019.

Preparation

1. As preparation for your own project work, formulate one or two project proposals that you will be willing to pitch when we meet for the synchronous part. The pitch should be 5 minutes and use 1 slide.

Assignment

2. Find a mathematical research paper, and choose a range of 5–10 pages from it. Identify and count all the verbs that occur in those pages.

3. Try to introduce some classification and formulate a hypothesis.

4. Write up a short (approximately 1 page) report presenting your quantitative data and your reflections and post it on `slack` by Friday. Include reflection on the robustness of your hypothesis given your data, and think about how you could go on extending your investigation. Please use the forum to engage with suggestions from others and develop ideas (both pre- and post-submit).

References for online week 4

Inglis, Matthew and Andrew Aberdein (2014). "Beauty Is Not Simplicity. An Analysis of Mathematicians' Proof Appraisals". *Philosophia Mathematica (3)*, vol. 23, no. 1, pp. 87–109. DOI: 10.1093/philmat/nku014.

Pease, Alison, Andrew Aberdein, and Ursula Martin (2019). "Explanation in Mathematical Conversations. An Empirical Investigation". *Philosophical Transactions A*, vol. 377, no. 2140. DOI: 10.1098/rsta.2018.0159.

Saris, Willem E. and Irmtraud N. Gallhofer (2014). *Design, Evaluation, and Analysis of Questionnaires for Survey Research*. 2nd ed. Wiley Series in Survey Methodology. Hoboken: John Wiley & Sons.

3   Synchronous part (August 10 to August 21)

Figure 2 provides an overview of the synchronous part of the course.

	Mo 10/8	Tu 11/8	We 12/8	Th 13/8	Fr 14/8		Mo 17/8	Tu 18/8	We 19/8	Th 20/8	Fr 21/8
A.M.	Intro-duction	L1	L2	L3	L4		Me-thod*	L* / C2	L* / C4	Work / Super-vision	Work
P.M.	Pitch	Work / super-vision	Work / super-vision	Work / Super-vision	Problem presen-tations	Weekend	L* / C1	L* / C3	L* / C5	L* / Work	Status presen-tations

Time slots: First week, morning sessions (AM) from 9.00 to 12.00 and afternoon sessions (PM) from 13.00 to 16.00. Second week, morning sessions (AM) from 9.00 to 12.00 and afternoon sessions (PM) from 14.00 to 17.00 with international guest lectures between 10.30 and 12.00 and between 15.30 and 17.00.

Introduction: Welcome to the synchronous part of the course; practical information; overview of PMP.

Pitch: Pitching of your project ideas, prepared in online week 4.

L1–L4: Lectures and discussion on the four themes (see below).

Work: Individual work on preparation lectures, presentations, conference and project.

Supervision: Scheduled supervision.

Problem presentations: Preliminary presentation of project proposals with peer-feedback.

Method*: Lecture and discussion on methodological questions relevant to your projects. Details to be decided during the first synchronous week.

L*: Lectures and discussions on topics of relevance to your projects. Details to be decided during the first synchronous week.

C1–C5: Invited online guest lectures for an informal mini-conference on TPMP with international researchers, followed by Q&A and discussion with the guest lecturers (see below).

Work: Individual work.

Status presentations: Mini-conference with presentations of your status and progress on yoru projects. BYOB!

Figure 2: Overview of the synchronous part.

3.1 Lectures

L1: Diagrams and cognitive artefacts

References for lecture L1

Johansen, Mikkel Willum, Morten Misfeldt, and Josefine Lomholt Pallavicini (2018). "A Typology of Mathematical Diagrams". In: *Diagrammatic Representation and Inference. 10th International Conference, Diagrams 2018. Edinburgh, UK, June 18–22, 2018*. Ed. by Peter Chapman, Gem Stapleton, Amirouche Moktefi, Sarah Perez-Kriz, and Francesco Bellucci. Lecture Notes in Computer Science 10871. Cham: Springer, pp. 105–119. DOI: `10.1007/978-3-319-91376-6_13`.

Shin, Sun-Joo, Oliver Lemon, and John Mumma (2014). "Diagrams". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Winter 2014. URL: `http://plato.stanford.edu/archives/win2014/entries/diagrams/`.

L2: Experimental methods in practice

References for lecture L2

Baker, Alan (2009). "Non-Deductive Methods in Mathematics". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Fall 2009. URL: http://plato.stanford.edu/archives/fall2009/entries/mathematics-nondeductive/.

Sørensen, Henrik Kragh (2010). "Exploratory experimentation in experimental mathematics. A glimpse at the PSLQ algorithm". In: *PhiMSAMP. Philosophy of Mathematics: Sociological Aspects and Mathematical Practice*. Ed. by Benedikt Löwe and Thomas Müller. Texts in Philosophy 11. London: College Publications, pp. 341–360. URL: http://www.lib.uni-bonn.de/PhiMSAMP/Book/.

L3: Writing and validating proofs

The lecture will treat proofs as vehicles of communication and center on Andersen (2018) and Andersen, Johansen, and Sørensen (2019) which you should browse and compare to e.g. Rav (1999) from the online part.

References for lecture L3

Andersen, Line Edslev (2018). "Acceptable gaps in mathematical proofs". *Synthese*. DOI: 10.1007/s11229-018-1778-8. Online first.

Andersen, Line Edslev, Mikkel Willum Johansen, and Henrik Kragh Sørensen (2019). "Mathematicians Writing for Mathematicians". *Synthese*. DOI: 10.1007/s11229-019-02145-5. Online first.

Rav, Yehuda (Feb. 1999). "Why do we prove theorems?" *Philosophia Mathematica (3)*, vol. 7, no. 1, pp. 5–41. DOI: 10.1093/philmat/7.1.5.

L4: Formal methods in practice

References for lecture L4

Geuvers, H. (2009). "Proof assistants. History, ideas and future". *Sādhanā*, vol. 34, no. 1, pp. 3–25. DOI: 10.1007/s12046-009-0001-5.

Johansen, Mikkel Willum and Henrik Kragh Sørensen (Sept. 2019). "Human and Computerized Mathematical Practice. Experimental Mathematics and Interactive Theorem Provers". Submitted.

3.2   Online mini-conference [preliminary schedule]

C1 Monday 17 Aug, 2020 from 15:30 to 17:00 (Danish time), **Keith Weber (Rutgers)** is invited to speak on connections between PMP and math.ed. based on the paper Juan Pablo Mejía-Ramos, Kristen Lew, Jimmy de la Torre, and Keith Weber (2017). "Developing and validating proof comprehension tests in undergraduate mathematics". *Research in Mathematics Education*, vol. 19, no. 2, pp. 130–146. DOI: 10.1080/14794802.2017.1325776.

C2 Tuesday 18 Aug, 2020 from 10:30 to 12:00 (Danish time), **Colin Jakob Rittberg (Amsterdam)** is invited to speak on epistemic injustice in mathematics based on the paper Colin Jakob Rittberg, Fenner Stanley Tanswell, and Jean Paul Van Bendegem (2018). "Epistemic Injustice in Mathematics". *Synthese*. DOI: 10.1007/s11229-018-01981-1.

C3 Tuesday 18 Aug, 2020 from 15:30 to 17:00 (Danish time), **Rebecca Lea Morris (Independent scholar)** is invited to speak on reward structures of mathematics based on the paper Rebecca Lea Morris (May 2020). "Intellectual generosity and the reward structure of mathematics". *Synthese*. DOI: `10.1007/s11229-020-02660-w`.

C4 Wednesday 19 Aug, 2020 from 10:30 to 12:00 (Danish time), **Fenner Tanswell (Loughborough)** is invited to speak on corpus analysis and imperatives in mathematical texts based on the paper Fenner Tanswell and Matthew Inglis (2020). "Actions and Imperatives in Mathematical Texts". Manuscript.

C5 Wednesday 19 Aug, 2020 from 15:30 to 17:00 (Danish time), **Zoe Ashton (Ohio State)** is invited to speak on mathematical audiences based on the paper Zoe Ashton (Mar. 2020). "Audience role in mathematical proof development". *Synthese*. DOI: `10.1007/s11229-020-02619-x`.

References for the mini-conference

Ashton, Zoe (Mar. 2020). "Audience role in mathematical proof development". *Synthese*. DOI: `10.1007/s11229-020-02619-x`.

Mejía-Ramos, Juan Pablo, Kristen Lew, Jimmy de la Torre, and Keith Weber (2017). "Developing and validating proof comprehension tests in undergraduate mathematics". *Research in Mathematics Education*, vol. 19, no. 2, pp. 130–146. DOI: `10.1080/14794802.2017.1325776`.

Morris, Rebecca Lea (May 2020). "Intellectual generosity and the reward structure of mathematics". *Synthese*. DOI: `10.1007/s11229-020-02660-w`.

Rittberg, Colin Jakob, Fenner Stanley Tanswell, and Jean Paul Van Bendegem (2018). "Epistemic Injustice in Mathematics". *Synthese*. DOI: `10.1007/s11229-018-01981-1`.

Tanswell, Fenner and Matthew Inglis (2020). "Actions and Imperatives in Mathematical Texts". Manuscript.

3.3 Student projects

In order to develop and prepare your individual projects, we will conduct four types of collaborative teaching: pitches, supervision, presentations, and a mini-conference.

Pitch

Before we meet for the synchronous part, students must prepare a short "pitch" of one (or two) project idea(s). The pitches will be presented on the first day of the synchronous part and will serve as introducing topics and for identifying possible collaborative projects.

Each pitch should last 7 minutes and you can rely on 1 (one!) slide.

Your pitch should include information on: 1. the topic you are interested in, 2. the reason(s) you find this topic interesting, 3. one question that you would be interested in researching (formulated as a question!), 4. the method you are interested in pursuing (case study, interview, quantitative methods), and 5. mention of the one (major) obstacle you can foresee in answering your question.

Supervision

During the synchronous part, you can book supervision (2 sessions of 30–45 minutes) with Mikkel or Henrik for your individual projects. Supervision can be conducted in groups where relevant. Please state before the session what would be the main focus of the session.

Problem presentations

By the end of the first synchronous week, we will have a round of work-in-progress presentations on the state of your projects.

Your presentation should be 10–15 minutes and must contain 1. information on your problem formulation, 2. introduction to your main source or set of data or methodology, and 3. information on your progress and your prioritization, i.e. what has been done and what needs to be done.

Status presentations

For the final day of the synchronous part, we will have a mini-conference with a series of your presentations on your progress and status. These presentations should be (more) formalized and report on your findings and (tentative) conclusions.

Your presentation should be 10–15 minutes and should be accessible to others who have not followed the course or your process. Think about what and how you present.

4   Exam: Project and report (due by August 28)

The final exam consists of two different parts: Your final answers to the four small, written assignments that are posted during the asynchronous on-line part of the course (July 13 to August 7) and a written project that is to be produced during the synchronous part of the course (August 10 to 21) and in the following week. All five parts of the assignment must be handed in no later than Friday, August 28, 2020.

4.1   The small, written assignments

During the (asynchronous) online part of the course we will cover four themes; one for each week. And each week we will pose a small assignment connected to that theme. You are supposed to post your preliminary answers to the assignments by the end of each week, and to discuss and comment constructively on the other students' answers in the following week such that you can help each other to improve ideas, content, and presentation. You are allowed to improve and rework your answer to the assignments throughout the course. Only the final version of your answers — the ones that you hand in as part of the deliverables at the end of the course — will be assessed by the teachers.

Notice that although we encourage students to share ideas and help each other to give better and more reflected answers to the assignments, the final answers to the assignments should be written individually. You are in other words not allowed to reproduce text written by other students, although it is only natural if your answers reflect and incorporate ideas and thoughts you have developed collectively during the course.

4.2   The written project

During the synchronous part of the course (August 10 to 21) and the following week you are supposed to pick a topic for and work with a larger, written project. The project should be written in the style of a research paper from the area of the philosophy of mathematical practice, and it should be no longer than 10 pages (2400 characters including spaces). You will have the opportunity to discuss the project with the other students during the course, and you will be offered individual supervision by the course teachers throughout the process.

You are welcome to collaborate on data collection as well as the written paper. If you collaborate on the written paper you must clearly indicate who is responsible for the different

sections of the paper; for each section, at least one person should be responsible, and each of the persons collaborating should be sole responsible for at least 7 pages of the paper.

The final version of the paper should be handed in along with the final version of your answers to the small assignments Friday August 28 via Digital Exam.

With the project you should demonstrate that you master the learning goals of the course (see below). When we assess the project, we will determine the extent to which this objective has been met.

4.3   Learning goals

After following the course students should have the following skills, knowledge and competences:

- Skills: The students should be able to

  – collect and analyze empirical material

- Knowledge: The students should be able to

  – account for central topics in the philosophy of mathematical practice.

- Competences: The students should be able to

  – reflect on the strengths and limitations of empirical methods.
  – discuss and reflect on central topics in philosophy of mathematical practice.
  – produce empirically informed written products in the area of philosophy of mathematics.