Chapter 1.

Introduction

An extract from

The mathematics of NIELS HENRIK ABEL.
Continuation and New Approaches in
Mathematics During the 1820s.

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CHAPTER 1

Introduction

In the aftermath of the French Revolution of 1789, the political and scientific scenes in Paris and throughout Europe underwent radical changes. Social and educational reforms introduced the first massive instruction in mathematics at the newly established *École polytechnique* in Paris; and mathematics, itself, changed and developed into a form recognizable to modern mathematicians. In the first decades of the 19th century, the neo-humanist movement greatly influenced Prussian academia and as an effect, mathematics was promoted into a very prominent position in the curriculum of secondary schools. At the university level, mathematics gained a certain autonomy and started to evolve along a distinctly theoretical line with less focus on applications and mathematical physics.

The present work centers on one of the main innovative figures in mathematics in the 1820s, the Norwegian NIELS HENRIK ABEL (1802–29), and describes his contribution to and influence on the fermentation of the mathematical discipline in the early 19th century. Born at the periphery of the mathematical world and with a life-span of less than 27 years, ABEL nevertheless contributed importantly to the disciplines which he studied. The overall outline of this presentation is recapitulated in the following three sections which introduce ABEL's professional background and training, the mathematics of his works, and the treated themes of development in mathematics in the first half of the 19th century. Throughout, ABEL's mathematics is seen in its mathematical context, and the influences of mathematicians such as CAUCHY, GAUSS, LAGRANGE, and LEGENDRE is traced and described. This approach provides a background for discussing aspects of continuity and transformation in mathematics as can be envisioned from ABEL's works.

1.1 The historical and geographical setting of ABEL's life

NIELS HENRIK ABEL lived in a politically turbulent time during which his birthplace, Finnøy, belonged to three different monarchies. When ABEL was born in 1802, Finnøy belonged to the Danish-Norwegian twin monarchy but in the wake of the Napoleonic Wars, the province of Norway was ceded to Sweden after a short spell of independence. Education in the twin monarchy was centered in Copenhagen, and only in 1813 was the university in Christiania (now Oslo) opened. The scientific climate was beginning to ripe, but mathematics was not studied at a high level.

As was common practice for the sons of a minister, ABEL attended grammar school in Christiania and soon got the young BERNT MICHAEL HOLMBOE (1795–1850) as a mathematics teacher. HOLMBOE was the first to notice ABEL's affinity for and skills in mathematics and they began to study the works of the masters in special private lessons. In 1821, after graduating from grammar school, ABEL enrolled at the university but continued his private studies of the masters of mathematics. In 1824, he applied for a travel grant to go to the Continent and he embarked on his European tour in 1825. It brought him to Berlin and Paris where he had the opportunities to meet some of the most prominent mathematicians of the time and frequent the well equipped continental libraries. More importantly, ABEL came into contact with AUGUST LEOPOLD CRELLE (1780–1855) in Berlin. CRELLE became ABEL's friend and published most of ABEL's works in the *Journal für die reine und angewandte Mathematik* which he founded in 1826. When ABEL returned to Norway in 1827 he found himself without a permanent job and with no family fortune to cover his expenses, he took up tutoring in mathematics. He had suffered from a lung infection during his tour, and in 1829 he succumbed to turberculosis.

ABEL's geographical background thus dictated his approach to mathematics; it forced him to study the masters and advance in isolation to do original work. In his short life span he carefully studied works of the previous generation and went beyond those. During the months abroad, he came into contact with the newest trends in mathematics, and was immediately engaged in new research. Almost all his publications were written during or after the tour. The presentation of ABEL's historical and biographical background serves to provide a framework for tracing ideas, influences, and connections in his work.

1.2 The mathematical topics involved

ABEL's mathematical production span a wide range of topics and theories which were important in the early 19th century. His primary contributions are universally considered to be in the theory of algebraic solvability of equations, in the rigorization of the theory of series, and in the study of elliptic functions and higher transcendentals. However, some of ABEL's other works (published or unpublished) also have their place in the contexts of other disciplines, e.g. in the solution of particular types of differential equations, in the prehistory of fractional calculus, in the theory of integral equations, or in the study of generating functions. However, to keep the focus of the present dissertation, these "minor" topics have not been included and emphasis is put on equations, series, and elliptic and higher transcendentals.

Theory of equations. The essentials of mathematics in the 18th century come down to the work of a single brilliant mind, LEONHARD EULER (1707–83). Through a lifelong devotion to mathematics which spanned most of the century preceding the French Revolution, EULER reformulated the core of mathematics in profound ways. Inspired by his attempts to demonstrate that any polynomial of degree n had n roots (the so-called Fundamental Theorem of Algebra), EULER introduced another important mathematical question: Can any root of a polynomial be expressed in the coefficients by radicals, i.e. by using only basic arithmetic and the extraction of roots? This question concerned the algebraic solvability of equations and to EULER it was almost self-evident. However, mathematicians strived to supply even the evident with proof, and JOSEPH LOUIS LAGRANGE (1736–1813) developed an elaborated theory of equations based on permutations to answer the question. Though a believer in generality in mathematics, LAGRANGE came to recognize that the effort required to solve just the general fifth degree equation might exceed the humanly possible. In LAGRANGE's native country, Italy, an even more radical perception of the problem had emerged; around the turn of the century, PAOLO RUFFINI (1765–1822) had made public his conviction that the general quintic equation could *not* be solved by radicals and provided his claim with lengthy proofs.

ABEL's first and lasting romance with mathematics was with this topic, the theory of equations; his first independent steps out of the shadows of the masters were unsuccessful ones when in 1821 he believed to have obtained a general solution formula for the quintic equation. Provoked by a request to elaborate his argument, he realized that it was in err, and by 1824 he gave a proof that no such solution formula could exist. The proof, which was based on a detailed theory of permutations and a classification of possible solutions, reached world (i.e. European) publicity in 1826 when it appeared in the first volume of CRELLE's *Journal* *für die reine und angewandte Mathematik*. But as so often happens, solving one question only leads to posing another. Realizing that the general fifth degree equation could not be solved by radicals, ABEL set out on a mission to investigate which equations could and which equations could not be solved algebraically. Despite his efforts — which were soon distracted to another subject — ABEL had to leave it to the younger French mathematician EVARISTE GALOIS (1811–32) to describe the criteria for algebraic solvability.

Elliptic functions. Since the emergence of the calculus towards the end of the 17$^{\text{th}}$ century, the mathematical discipline of analysis had been able to treat an increasing number of curves. In his textbook *Introductio in analysin infinitorum* of 1748, EULER elevated the concept of function to the central object of analysis. Concrete functions were studied through their power series expansions and the brilliant calculator EULER obtained series expansions for all known functions including the trigonometric and exponential ones. However, EULER did not stop there but ventured into the territory of unknown functions of which he tried to get hold. One important type of function which analysis had struggled to treat on a par with the rest was the so-called elliptic integrals which can measure the length of an arc of an ellipse.

Mathematicians such as EULER and ADRIEN-MARIE LEGENDRE (1752–1833) felt and spoke of an unsatisfactory restriction of analysis because it was only able to treat a limited set of elementary transcendental functions. Admitting new functions into analysis meant obtaining the kind of knowledge about these functions that would allow them to be given as *answers*. If a function today is nothing more than a mapping of one set into another, the knowledge of a function then included tabulation of values, series expansions and other representations, differential and integral relations, functional relations, and much more.

When ABEL made elliptic integrals his main research topic, much knowledge concerning these objects had already been established. An algebraic approach which had profound influence on ABEL was GAUSS' study of the division problem for the circle (construction of regular n-gons) in the *Disquisitiones arithmeticae*.[1] GAUSS had hinted that his approach could be applied to the lemniscate integral, a particularly simple case of elliptic integrals, and ABEL took it upon himself to provide the claim with a proof. By a new idea, soon to be praised as one of the greatest in analysis, ABEL inverted the study of elliptic integrals into the study of elliptic functions: Instead of considering the value of an integral to be a function of its upper limit, he considered the upper limit to be a function of the value of the integral (compare arcsin and sin). Through formal substitutions and certain addition formulae, ABEL obtained elliptic functions of a complex variable. By this

[1](Gauss 1801).

inversion of focus, ABEL managed to place the entire theory of elliptic integrals on a new and much more fertile footing. Fueled by a fierce competition between ABEL and the German mathematician CARL GUSTAV JACOB JACOBI (1804–51), the new theory gained almost immediate momentum and became one of the central pillars of and main motivations for 19^{th} century advances in mathematics.

Although ABEL had presented the crucial idea of inverting elliptic integrals into elliptic functions, his impact on the further development of the theory stemmed as much from a vast generalization of the addition formulae presented in a paper which he handed in to the Parisian *Académie des sciences* in 1826 (not published until 1841). In this paper, ABEL treated an even broader class of integrals generalizing the elliptic ones and — again using primarily algebraic methods — proved more general versions of the addition theorems. The quest of later mathematicians to reapply ABEL's daring inversion of elliptic integrals to this broader class of integrals led to much of the important development in complex analysis and topology in the 19^{th} century.

Rigor. Although the theory of equations was closest to ABEL's heart, and the theory of elliptic functions brought him fame in the 19^{th} century, his mathematical legacy remembered in the 20^{th} century is just as much about his intense reception of CAUCHY's new rigor. Picking up from LAGRANGE's theory of functions, CAUCHY had placed concepts such as continuity and convergence in the foreground and founded these concepts on a new interpretation of *limits*. Equally importantly, CAUCHY had shown a way of working with these concepts to deduce properties of *classes* of objects (e.g.. continuous functions or convergent series) rather than explicit, often lengthy, studies of specific objects.

In a memorable and often quoted letter dated 1826 (first published 1839), ABEL expressed his conversion to CAUCHY-ism and gave the new rigor its dogmatic manifesto. Apparently more radical than CAUCHY himself, ABEL helped determine the formulation of the new rigor through his interpretative readings of CAUCHY. In the process of refounding analysis on rigorous grounds, central concepts were specified and *changed* (stretched) to an extent where they included elements whose behavior was deemed abnormal. The encounter and resolution of these abnormalities, *exceptions* as they were often called, was an integrated part of the rigorization process; such exceptions — which a modern reader would consider *counter examples* — shed interesting light on the role and use of concepts in mathematics in the early 19^{th} century.

1.3 Themes from early 19th century mathematics

The early 19th century marks a period of transition and fermentation in mathematics which involves most layers of the discipline, external as well as internal. With the boundaries fixed, say, between 1790 and 1840, a definite change in the way mathematics was performed and presented is evident; research mathematicians began working in institutions set up for instruction in mathematics and started presenting their results in professional periodicals with substantial circulation. However, the change even effected the internal core of the discipline: how mathematics was done, what mathematics was, and which mathematical questions were interesting. Gradually, *concepts* and relations between concepts took an increasingly central position in mathematics research; although the concern for concrete objects never ceased completely.

Concept based mathematics. Concepts such as *function*, *continuity* of functions, *irreducibility* of equations, and *convergence* of series attained central importance in mathematical research in the transitional period. CAUCHY's contribution to the rigorization of the calculus laid as much in *applying* technical definitions of concepts to prove theorems as with providing the definitions, themselves. Generalization in the 1820s turned the attention from specific objects to *classes* of objects, which were then investigated. This shift of attention towards collections of individual objects had a very direct influence on the style of presenting mathematical research. In the 'old' tradition, mathematical papers could easily be concerned with explicit derivations (calculations) pertaining to single mathematical objects. Although this presentational style far from ceased to fill periodicals, a less explicit style gained impetus in the first half of the 19th century. By deriving properties of classes instead of individual objects, the arguments became more abstract and often more comprehensible by lowering the load of calculations and simplifying the mathematical notation. The transition is evident in ABEL's works which show deep traces of the calculation based approach to doing mathematics as well as being markedly conceptual at times; his 1826 paper on the binomial theorem is a fascinating mixture of both approaches.

Abstract definitions and coming to know mathematical objects. In many developing fields of mathematics in the early 19th century, new concepts were specified by the use of abstract definitions based on previous proofs, intentions, and intuitions. In the approach which I term *concept based mathematics*, the concepts were *defined* in the modern sense that there is nothing more to a concept than its definition. However, when abstract definitions determine the extent of a concept, representations and demarcation criteria are required in order to get hold of properties of objects, and this quest for understanding, *coming to know*, the objects is an important aspect of early 19th century mathematics. In many ways, analogies may be drawn to the effort of coming to know geometrical objects, e.g. *curves*, in the 17th century. To mathematicians of the 17th century, a curve meant more than any single given piece of information. In particular, an equation (or a method of constructing any number of points on the curve) was not considered sufficient to accept the curve as *known*. Similarly, in the 19th century, knowledge of an elliptic function meant more than just a formal definition and included various representations, basic properties, and even tabulation of values.

The question of coming to know a mathematical object relates to the problem of accepting the object as solutions to problems. The reduction of properties of curves to questions pertaining those *basic* curves which were considered well known was important in the 17th century. However, certain properties were not expressible in basic curves (or functions) but required higher transcendentals such as elliptic integrals. Thus, much of EULER's research on elliptic integrals in the 18th century can be seen as an effort to make these integrals *basic* in the sense of acceptable solutions to problems. This research program was continued and reformulated in the 19th century during which the foundations, definitions, and framework of elliptic functions underwent repeated revolutions.

Critical revision. The critical mode of thought, rooted in the Enlightenment, had a profound impact on mathematics. Together with the demand for wider instruction in mathematics, the critical attitude brought about a deeply sceptical reading of the masters which focused on the foundations. In geometry, some mathematicians began to believe in the possibility of a non-Euclidean version, and in analysis, the long-standing problem of the foundation of the calculus was made an important *mathematical* research topic.[2]

CAUCHY's definition of the central concept of limits was itself a novelty, but of equal importance was the outlook for a concept based version of the calculus. CAUCHY's new foundation for the calculus was *arithmetical* and introduced the arithmetical concept of equality. In the wake of the change of foundations of the calculus, certain objects and methods could no longer be allowed into analysis, and it became a quest to prop up parts of the mathematical complex recently made insecure. In particular, CAUCHY had to abolish from analysis all divergent series which had formerly been interpreted by a *formal* concept of equality. However, divergent series had provided new insights to mathematicians which they were reluctant to abandon and it became a legitimate, albeit difficult, mathematical problem to investigate how problematic or outright unjust procedures had led to correct results. Resolution of this problem laid in further specification of concepts involved and a heightened awareness of the procedures employed in

[2]See e.g. (Grabiner 1981).

arguments. For instance, by the mid-19th century, the unreflective interchange of orders of limit processes had been identified as problematic and concepts such as absolute and uniform convergence had been introduced and put to use in theorems and proofs.

1.4 Reflections on methodology

The present study aims at illustrating important conceptual developments in mathematics which took place in the first decades of the 19th century. In order to introduce a focus on the many-faceted aspects of these developments, the mathematical production of NIELS HENRIK ABEL has been taken as a starting point. In the present section, some of the methodological choices and considerations involved in the project are briefly discussed. It is not my ambition to present a coherent theoretical framework for historical enquiries but rather to make some considerations explicit and open for discussion.

1.4.1 Diachronical descriptions

It is within the framework of mathematics developed by EULER and cultivated in the 18th century that ABEL's production is rightfully seen. Although his mathematics has been perceived as a very important step *forward* in a linear development, ABEL's mathematical ideas were rooted in the previous attitude and style; and many of the famed new trends are only barely recognizable in his work. Therefore, anachronisms and teleological conceptions have to be dismissed in favor of a diachronical, more hermeneutic approach. In the process of tracing and describing this historical evolution of mathematical content, it is of the utmost importance that ABEL's works be studied within their contemporary framework — their mathematical context.

Each of the main theories outlined above with which ABEL was involved is reviewed with the purpose of illustrating how they were effected by currents of mathematical change in the early 19th century. To do so, ABEL's mathematics is presented and discussed based on a contextualized reading which emphasizes ABEL's own methods and tools. To place these in their proper historical contexts, the theories and results will be traced back into the 18th century in search of the inspirations and their progressions in the 19th century will be followed. In the theory of equations, for instance, the works of EULER on the fundamental theorem of algebra will be briefly introduced; more emphasis will be given to the works of LAGRANGE and GAUSS which served as the direct inspirations for ABEL and most of his contemporaries in the field. Then, in the 19th century, special emphasis is given to those works which share their inspirations with the works of ABEL, in this case the works of RUFFINI and GALOIS. For each disciplinary theme, a sketch of the further development after the initial decades of the 19th century is then given in order to illustrate how the ideas and currents which were barely discernible in the first decades came to play very important roles in the conceptions of mathematicians. The descriptions of the ensuing histories also serve to illustrate how ABEL's works were valued and received by the following generations.

Besides, it has been a secondary aim of the present study to make ABEL's authentic mathematical thought available to the mathematically trained reader who is not familiar with early 19th century technicalities. In order to understand and evaluate ABEL's role in the formation of modern mathematics, this presentation will always favor the original source over any modern approach.

The setting of ABEL's mathematics within the general view of mathematics expressed by EULER will also be manifest in another way as a chronological mark. It has been necessary to trace many of the ideas and methods of ABEL's mathematics back to the middle of the 18th century, but they might be even older. However, as this is a work on ABEL's mathematics, such hypothesis will rarely be made explicit and EULER will be attributed things, which he did do — perhaps not as the first.

1.4.2 Philosophical theories and their applicability

Philosophical theories enter the framework of the present study only rather implicitly. Written with the utopian goal of being an "account of things which happened", the outlines of a certain perception of concepts such as *change* and *transformation* is nevertheless discernible. The internal (and external) structure of scientific change has been subjected to many philosophical investigations over the past decades. In the present context, however, two of the founding theories have served as inspirations; those of *Kuhnian paradigms and revolutions* developed for the sciences in general and those of *Lakatosian dialectics* which were developed explicitly for mathematics and illustrated by examples from the 19th century. These two philosophical positions are so well established within the history of science community that only a brief presentation is given with an emphasis on their applicability to the present study.

KUHN, paradigms, crises, and revolutions. In his very influential monograph *The structure of scientific revolutions* of 1962,[3] THOMAS KUHN (1922–1996) advocated an explanation of the dynamics of scientific change. The total mental and physical entourage of a science at a given time was encompassed in the notion of

[3](Kuhn 1962)

paradigms. Paradigms are abruptly replaced through *revolutions* which are the responses to *crises* brought about by a compilation of *anomalies* inexplicable within the ruling paradigm. Once a revolution has taken place, a new paradigm is introduced and communication between two distinct paradigms (e.g. over which paradigm to prefer) becomes irrational or extra-scientific. KUHN's original model — although deeper than the present description — was a simplistic one which was amended and extended by numerous studies following its publication. Here, on the other hand, it will not be taken to serve as a complete model but rather as inspiration and terminological framework used to capture important aspects.

As a model of mathematical change in the early 19th century, the Kuhnian system offers some obvious advantages; the important position given to anomalies in bringing about crises and revolutions was further extended in the works of IMRE LAKATOS (see below). However, as has been emphasized by many philosophers and historians of mathematics, no overthrow of knowledge seems to occur in mathematics; thus no truly Kuhnian revolutions seem possible in the mathematical realm.[4] The remaining notions of the Kuhnian conceptual framework such as paradigms, anomalies and crises are, however, applicable and useful in the description and analysis of mathematical change, even if KUHN's dynamics are not always appropriate. In the first sections of part **??**, for instance, it is illustrated how a mathematical theory came into being by a change of focus (a paradigmatic change) which shifted emphasis to questions of solvability. The theme will recur even more distinctly in part **??** which documents ABEL's role and position in the most Kuhnian of changes in mathematics during the early 19th century: the complete reformulation of analysis according to CAUCHY's new program of arithmetical rigor.

LAKATOS and the extension of concepts. Further philosophical inspiration is taken from IMRE LAKATOS' (1922–1974) *Proofs and Refutations* published as a series of articles in 1963–64 and as a book in 1976.[5] LAKATOS described the dynamics of mathematical change in terms of a dialectic between *proofs* and *counter examples* by means of *proof revisions*. In the main part of the *Proofs and Refutations*, LAKATOS explained his theory by exhibiting a rationally reconstructed development of the Eulerian polyhedral formula; in appendices, he further illustrated the theory by exhibiting applications to other concepts including the development of the concept of uniform convergence (see part **??**).

LAKATOS saw the process of proof as central to the mathematical endeavor. Incorporating into mathematics a version of POPPER's falsificationism, LAKATOS described mathematical change as a continued revision of proofs to reflect objec-

[4]See (Gillies 1992).

[5](Lakatos 1976). A good description of LAKATOS' life and philosophy can be found in (Larvor 1998).

tions raised by counter examples. LAKATOS classified counter examples as either local (refuting only part of a proof, but not the overall statement) or global (refuting the overall statement, but not necessarily any identifiable part of the proof).

Counter examples could, in LAKATOS' description, be constructed from existing proofs by a process of *concept stretching* by which a partially defined concept was redefined in an extended version which — although possibly more precise — encompassed instances (objects) not covered by the previous — often more intuitive — version of the concept.

In response to such falsifications (refutations) by counter examples, LAKATOS suggested various strategies for refining the proofs. A naive approach would try to explain the counter examples away, either by arguing that they were too pathological to be taken seriously or (more interestingly) by restricting the theorem to a narrower domain for which it was believed to surely valid; the latter approach was named *exception barring* by LAKATOS. A more fruitful response to the refutation by counter examples — and the one which LAKATOS' philosophy dogmatized — was the method of *proof analysis* which took the counter examples more seriously. By carefully analyzing the counter example and the proof which it refuted, proof analysis produced a new proof in which a refuted lemma was replaced by an unrefuted one which might cause an alteration of the overall statement. Thus, LAKATOS suggested, theorems were produced which had very explicit assumptions and were very hard to refute.

Just as was the case with the Kuhnian model, LAKATOS' model — in all its generality — is often found inadequate to describe the actual historical development of mathematics. On the other hand, LAKATOS' model offers some further concepts which often ease the description and analysis of past events. Most importantly, LAKATOS' description of counter examples and the role that they play in mathematical change elaborates the role played by anomalies in the Kuhnian model and suggests a more refined view on the status of a mathematical theory in crisis.

The Lakatosian theory of mathematical evolution is present as background throughout; it will surface sporadically in parts **??**–**??** and become important again in the final, more analytical part **??**.

EPPLE's epistemic configurations. Quite recently, MORITZ EPPLE has suggested the notion of *epistemic configurations* in order to be able to discuss change in mathematics in another context.[6] In EPPLE's analysis, epistemic configurations consist of *epistemic objects* and *epistemic techniques* and are manipulated in mathematical *workshops*. The concept of epistemic objects encompasses the immaterial objects with which mathematics deals. These are manipulated and investigated by a

[6](Epple 2000).

number of methods of producing (or obtaining) mathematical knowledge; these methods are the epistemic techniques. The precise applicability and range of EPPLE's concepts and their usefulness in historical analysis is not the primary objective here. Instead, as with the inspirations of KUHN and LAKATOS, I have taken the liberty of using EPPLE's terms to ease the analysis and discussion of what I believe to be a fundamental change in mathematics in the early 19th century: the change from *formula based* to *concept based* mathematics which is addressed in chapter **??**.

1.4.3 Existing literature

Being one of the important mathematicians of the 19th century, ABEL's person and his mathematics have been subjected to study for a multitude of different reasons. A few general trends of the literature on ABEL can profitably be identified at this point.[7] At the relevant places in the subsequent parts, references are given to the secondary literature which is listed in the bibliography.

ABEL in the history of mathematics literature. In the professional literature in the history of mathematics, ABEL is often mentioned in order to illustrate one or more of the following aspects:

1. ABEL's life story is invoked to illustrate the conditions of young mathematicians two centuries ago. This aspect is closely related to the biographies treated below.

2. ABEL's letters from Paris are used to illuminate how the confrontation with CAUCHY's new rigor brought about a radical change. For instance, UMBERTO BOTTAZZINI quotes *in extenso* from these letters in his comprehensive account of the evolution of analysis in the 19th century.[8]

3. The modern highlights of ABEL's production, e.g. the binomial theorem or the unsolvability of the quintic, are described to shed some light on the evolution of the theories and the involved concepts.

4. ABEL's mathematics is described *per se* in order to give a presentation of his production. Very good examples include articles by SYLOW in the ABEL centennial memorial volume and the second edition of the collected works.[9]

The present study incorporates all these approaches to give a comprehensive overview of ABEL's mathematical production as well as positioning it within a broader frame describing themes of mathematical change in the period.

[7] For a thematic listing of the ABEL literature, see also (Sørensen 2002, to appear).

[8] (Bottazzini 1986)

[9] (Sylow 1902; Abel *Œuvres*$_2$)

Two other types of studies treating the life and works of ABEL delineate themselves: biographies and interpretations.

Biographies — scientific or not. As should become clear in the next chapter, ABEL's biography includes all the components of a truly romantic biography of a misunderstood genius who rose from the dust to become a nobility of mathematics. Such biographies have been written;[10] but more interestingly, biographies have also been written which serve a purpose of their own. The first biographies appeared as obituaries written by ABEL's friends soon after his death. Of primary importance in describing ABEL's mathematics are the obituaries written by HOLMBOE and CRELLE which include first hand descriptions of ABEL's mathematical work.[11] Although a larger number of biographies could be listed, the most widely circulated and very well researched 20th century biography was written by ØYSTEIN ORE;[12] it has been used mainly to help set the chronology straight. The human and cultural aspects of ABEL's life has most recently been very carefully researched and described by ARILD STUBHAUG who meticulously sets the cultural scene of early 19th century Norway and Europe.[13] STUBHAUG's biography has relieved me of any obligation to produce biographical news concerning ABEL's person; the biography which is provided in chapter **??** serves merely to set the framework of the subsequent chapters. It is my hope that the present study of ABEL's mathematics will complement STUBHAUG's book on his life and environment to produce a picture of ABEL's person *and* his mathematics.

Renderings of ABEL's work in modern theories. By the very nature of mathematics, mathematical knowledge seems to accumulate and only change its presentational form or its internal relations within mathematical structures. For this reason, mathematicians often hope to find inspiration in the works of their predecessors. Frequently, this leads to the publication of modernized versions of historical proofs. By itself, this practice is very good as long as the author and the community recognize that it is precisely a revisited proof or theorem and precisely *not* a diachronical description of that proof or theorem within its contemporary structure.

Such revisits to ABEL's production are most frequently made to his theory of algebraic solvability of equations, more precisely to his proof of the unsolvability of the quintic equation.[14] ABEL's other main contributions attract less attention; the binomial theorem because it has become an integral part of basic mathemati-

[10] E.g. (Bell 1953).
[11] (Crelle 1829; Holmboe 1829)
[12] (Ore 1954; Ore 1957)
[13] (Stubhaug 1996; Stubhaug 2000)
[14] See e.g. (Ayoub 1982; Radloff 1998).

cal knowledge, and the *Abelian theorem* (see part **??**) because its original form has been surpassed and the result has been recast in a different theory.

As should now be clear, the methodology of the present approach can be summarized thus: A diachronical reading of the original sources of ABEL's mathematics with the purpose of analyzing themes of mathematical change in the early 19th century, in particular the rise of concept based mathematics.

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